SIMON'S CLAIM FOR GAUSS IN NON-EUCLIDEAN GEOMETRY.

By DR. GEORGE BRUCE HALSTED, Gambier, Ohio.

In Vol. X, of THE AMERICAN MATHEMATICAL MONTHLY, p. 186, foot-note, Professor J. W. A. Young quotes, with apparent acceptance, a legendary assertion of Herr Simon. This assertion is the triple blunder that there is an "established recognition" by Gauss "about 1792" that the parallel axiom is indemonstrable; and that Gauss "influenced" John Bolyai and Lobachevski, the real founders of non-Euclidean Geometry.

One of the very greatest creations in mathematics since ever the world began is, beyond peradventure, the non-Euclidean geometry.

By whom was this given to the world in print?

By a Magyar, Bolyai János, who created it in 1823, and by a Russian, Lobachevski, who had made the discovery by 1826.

Were either of these men influenced, prompted, helped, or incited by Gauss, or by any suggestion emanating from Gauss?

No; quite the contrarv. Our warrant for saying this with final and overwhelming authority is the eighth volume of Gauss's own works, first published in 1900, where with great minuteness every scrap is published which could, by any interpretation, connect Gauss with the great Bolyai-Lobachevski creation.

The geometric part opens, p. 159, with Gauss's letter of 1799 to Bolyai Farkas the father of John (Bolyai János), which I gave in 1896 in my Bolyai (Vol. 3 of the Neomonic. Series) as demonstrative evidence that in 1799 Gauss was still trying to prove Euclid's the only non-contradictory system of geometry, and also the system of objective space.

The first is false; the second can never be proven.

So far was Gauss even then from recognizing that the parallel axiom is not a logical necessity, that both he and his friend kept right on working away to prove it a logical necessity, and the more hot-headed of the two, Farkas, finally thought he had succeeded, and in 1804 sent his proof to Gauss, in his "Göttingen Theory of Parallels."

Gauss's judgment on this is the next thing given (pp. 160-162). He shows the weak spot, saying: "Could you *prove*, that $dkc=ckf=fkg$, etc., then were the thing perfect. However, *this theorem is indeed true*, only difficult, without already presupposing the theory of parallels, to prove rigorously."

These words knock out forever the claim made by Simon for Gauss.

Thus in 1804, instead of having or giving any light, Gauss writes that the only link missing in his friend's attempt to prove the parallel axiom a logical necessity is true, though difficult to establish without petitio principii.

Of course since Bolyai János and Lobachevski all the world now knows it is impossible to prove, knows that Gauss was mistaken. Yet both the friends continue their strivings after this impossibility.

In this very letter Gauss says: "I have indeed yet ever the hope that those

rocks sometime, and indeed before my end, will allow a thorough passage."

Farkas on December 27, 1808, writes to Gauss: "Oft thought I, gladly would I, as Jacob for Rachel serve, in order to know the parallels founded even if by another." "Now just as I thought it out on Christmas night, while the Catholics were celebrating the birth of the Saviour in the neighboring church, yesterday wrote it down, I send it to you enclosed herewith." "Tomorrow must I journey out to my land, have no time to revise; neglect I it now, may be a year is lost, or indeed find I the fault, and send it not, as has already happened with hundreds, which I as I found them took for genuine. Yet it did not come to writing those down, probably because they were too long, too difficult, too artificial; but the present I wrote off at once. As soon as you can, write me your real judgment."

This letter Gauss never answered, and never wrote again until 1832, a quarter of a century later, when the non-Euclidean geometry had been published by both Lobachevski and Bolyai János.

This settles now forever all question of Gauss having been of the slightest or remotest help or aid to young János, who in 1823 announced to his father Farkas in a letter still extant, which I saw in Maros-Vásárhely, his creation of the non-Euclidean geometry as something undreamed of in the world before.

This immortal letter, a charming and glorious outpouring of pure young genius, I gave in the Introduction to my *Bolyai*, 1896. It was reproduced in fac simile as frontispiece to the Bolyai Memorial Volume in 1902.

The equally complete freedom of Lobachevski from the slighest idea that Gauss had ever meditated anything different from the rest of the world on the matter of the parallel axiom I showed in *Science*, Vol. IX, No. 232, pp. 813-817.

Of two utterly worthless theories of parallels Gauss gave extended notices in the Göttingische gelehrte Anzeigen.

To Lobachevski's Theory of Parallels, to John Bolyai's marvelous Science Absolute of Space, Gauss vouchsafed never one printed word.

As Staeckel gently remarks, this certainly contributed thereto, that the worth of this mathematical gem was first recognized when János had long since finished his earthly career.

A PROOF THAT FOUR LINES IN SPACE ARE IN GENERAL MET BY TWO OTHER LINES.

By DR. T. M. PUTNAM, University of California.

Using the ordinary method of descriptive geometry a point in space is represented by *two* points in a plane, viz., by its horizontal and vertical projections, the vertical plane being thought of as rotated into coincidence with the horizon-